A Heuristic for the Vehicle Routing Problem with Tight Time Windows and Limited Working Times

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Abstract

The Vehicle Routing Problem with Time Windows (VRPTW) is a well-studied capacitated vehicle routing problem where the objective is to determine a set of feasible routes for a fleet of vehicles, in order to serve a set of customers with specified time windows. The ultimate optimization objective is to minimize the total travelling time of the vehicles. This paper presents a new hybrid heuristic approach for the problem and explores ideas for improvement. The proposed heuristic was tested on both small and large-scale problems, and whenever possible, results were compared to an available mathematical model implemented in OPL/CPLEX (Yang, 2008). Experiments show that the heuristic performs well in terms of both solution quality and computation time, making it especially valuable for larger problems where the mathematical model is unable to produce solutions in reasonable time.

Keywords: Hybrid Heuristic for VRPTW, Tight Time Windows, Limited Working Times

1. Introduction

Vehicle routing problems (VRPs) are difficult optimization problems where there is a need to find prompt feasible and efficient solutions. Briefly, VRP is the determination of an optimal set of routes for a fleet of vehicles in order to serve a given set of customers. It is one of the most important and well-studied combinatorial problems in optimization literature (Toth and Vigo, 2002). Additionally, the objective of VRPTW is to serve a number of customers that have predefined time windows at minimum cost (in terms of distance travelled), without violating the capacity and total working time constraints of each vehicle (Tan, et al., 2001). Therefore, VRP receives considerable amount of attention from industries and has become a central problem in the field of transportation, distribution and logistics (Vidal et. al., 2013). In this paper, a new hybrid heuristic approach for VRPTW is proposed by simultaneously considering clustering and savings algorithms where customers are clustered and segmented based on their time-windows. Also, driving times and working times are individually considered. Our proposed hybrid heuristic is compared with the previously available mathematical models in the literature. The organization of the paper is as follows; Section 2 briefly discusses the literature about heuristics and the VRP. Section 3 details the problem description, Section 4 describes the heuristic, Section 5 presents the experimentation results and a brief discussion on the results, and Section 6 gives a review of conclusions and potential areas of future research.

2. Literature Review

Since VRP is known to be an NP-hard problem in the strong sense and it cannot be solved optimality in polynomial time, researchers have resorted to developing heuristic algorithms to in the last 40 years. These heuristics can obtain reasonably good feasible solutions in a relatively short time. These approaches provide fast solutions however, they cannot guarantee optimal solutions. Heuristic approaches that have been proposed for solving VRP problems, are classified mainly into three categories. These are; classical heuristics; that are developed between 1960's and 1990's, improvement heuristics; that are usually generated by construction heuristics for improving the solution quality and meta-heuristics; that are being developed since 1990's. Several heuristics are presented in this section to show the evaluation of VRP heuristics in the literature (Laporte et al., 2000).

In the literature, the simplest yet most powerful studied member of VRP family is the *capacitated* VRP *(CVRP).* In the CVRP, a fleet of identical vehicles located at a central depot is optimally routed to cover a set of customers with known demands. Each vehicle can perform at most one route and the total demand of the customers visited by a route cannot exceed the vehicle capacity. Along with that, another important variation of *VRP* problems is the VRP with *time windows (VRPTW)* that generalizes the CVRP by imposing that each customer is visited within a specified time period, called *time window(Toth and Vigo, 2014).*

Classical/construction heuristics can be considered as the most well-known optimization strategies for solving combinatorial optimization. These approaches start from null-solution and generate feasible solutions by achieving simple steps. These approaches are continued until a complete solution is achieved and termination criteria are met. All in all, these construction heuristics find one feasible solutions quickly whereas these feasible solutions may have large gap in comparison to the best solution (Sedighpour et al., 2011).

In 1964, Clarke & White's Savings Algorithm is proposed in which the heuristic is designed as follows; goods from a single source depot must be delivered in given quantities to given customers. The overarching goal of this algorithm is to determine the allocation of the customers among routes, and determine the sequence in which the customers are visited on a route, and which vehicle that should cover a route. The ultimate objective is to find the total transportation costs. Beasley (1983) proposed the cluster first, route second approach. In this approach, the customers are clustered based on their coordinates and then a nearest neighbor search algorithm is employed where a customer is connected to the nearest one until the termination criteria are met.

Solomon (1987) proposed a minimum cost of insertion technique. In this technique, after clustering is performed and feasible tours are constructed, a decision is made whether switching a customer from one tour to another is advantageous in terms of distance or cost. Gillet and Miller (1974) proposed a sweep algorithm consists of two stages, clustering and route generation. At the clustering stage, all nodes are clustered based on their capacity. In goods delivery vehicle routing, capacity is the maximum number of goods that can be carried in serving a route. In this case, the maximum number of goods carried by the vehicle depends on the capacity of the vehicle itself. The Fisher and Jaikumar (1981) algorithm is well-known cluster-first, route-second algorithm. Instead of using a geometric method to form the clusters, it solves a Generalized Assignment Problem (GAP) which consists of four steps: seed selection, allocation of customers to seeds, allocation of customers to seeds, generalized assignment and a traveling salesman problem (TSP) solution.

Bramel and Simchi-Levi (1995) developed a two-phase heuristic in which the seeds are determined by solving capacitated location problem at first. Then in the second stage; remaining vertices are gradually allocated into their corresponding routes. In this research initial seeding was performed by considering the minimum total distance of customers to the closest seed while considering the assigned total demand does not exceed the vehicle capacity. Then, vehicle routes are generated by inserting each customer with a minimum insertion cost. Moreover, Renaud et al.,(1996a; 1996b) developed petal algorithms which are the extensions of sweep algorithms that consists of construction of an initial envelope, insertion of the remaining vertices, and improvement procedure. Briefly, several routes are generated called *petals* and final decision is made by solving a set portioning problem.

Although in 2000's, meta-heuristics are widely applied to solve VRPs with time windows constraints, several heuristics were also developed to find near-optimal solutions. Dullaert et al., (2002) extended Solomon's (1987) sequential insertion heuristic with vehicle insertion savings with by considering heterogeneous fleet sizes with mix vehicle routing decisions. The vehicles were different in terms of equipment, carrying capacity, age, cost and addresses to different market segments. Moreover, Funke et al., (2005) presented a *local search* method for VRPs to find a local best neighbor and to reach a local optimum as quickly as possible. Briefly, the researchers defined the neighborhoods by a set of moves and showed how moves can be decomposed further into partial moves. The search method composes these partial moves into complete moves in quickest way. Irnich et al., (2006) extended their previous research by incorporating *sequential search technique* in their local search method. In their search technique, neighborhoods are investigated in an efficient way and the intention of the paper was to present a *sequential search* as a generic technique in VRPs. The sequential search was compared to traditional algorithms for the exploration of the neighborhood, which can categorize as lexicographic search approaches. Recently, Jokar and Sahraeian (2012) developed greedy heuristic algorithm for location routing problem with multiple depots. A heuristic approach was developed and embedded to simulated-annealing. The purpose of this research was to decide which customers are served by which depots and the routes of customers from the corresponding depots. The overview of aforementioned heuristics can be found in Laporte and Semet (2002), Pureza et. al., (2012) and Vidal et al., (2013). Vidal et al., (2013) pointed out that, the aforementioned heuristics were capable of producing solutions within 10%-15% of the optimum within a short period of time. Recently various meta-heuristic approaches are used those heuristics to fortify their solution quality. This paper will be unique in the sense of developing *hybrid* heuristic that encompasses a simultaneous consideration of clustering and savings algorithms where customers are clustered and segmented based on their time-window. In the meantime, driving times and working times are individually considered.

3. Problem description

The Vehicle Routing Problem has many variations, each aiming to deal with a specific problem of interest in the industry. Here is a list of assumptions made for the version used in this paper:

1- There is only one depot.
2- There is no working time limitation for the depot.
3- Each customer is visited exactly once, meeting all of its demand in whole.
4- Each customer has a specific service time window, outside which service is impossible.
5- Vehicles are all identical, with the same capacity.
6- Drivers are all identical, with the same driving time and working time limitations.
7- Vehicle capacity is at least as high as the largest customer demand.

In addition to the driving time, each customer has a specific service time that is added to the working time of the vehicle visiting it. The heuristic must be able to at least find a feasible solution based on the above limitations, and if there is more than one feasible solution, find one of the better ones. This makes the heuristic particularly interesting compared to simpler cases where there is no time window constrains. Figure 1 illustrates a sample problem with 8 customers. Note that customers are numbered and number 1 is reserved for the depot. Driving time and working time limitations used for all problems in this paper are 10 and 14 hours respectively, and vehicle capacity is 36 and average vehicle speed is 75 miles per hour. Figure 2 shows the optimal routing found for the sample problem by the mathematical model[1]. Note that these routes are directed (contrary to the routes for a VRP without time windows), and it is easy to see which direction is the correct one by looking at the time windows. For example routes going through 4, 6 and 7 goes first to 4, then 7 and then 6, because it is impossible to go the other way, considering that the start of customer 6 time window is later than the end of those of customers 4 and 7.

[1] IBM ILOG CPLEX Version 12.6 was used for mathematical analysis.

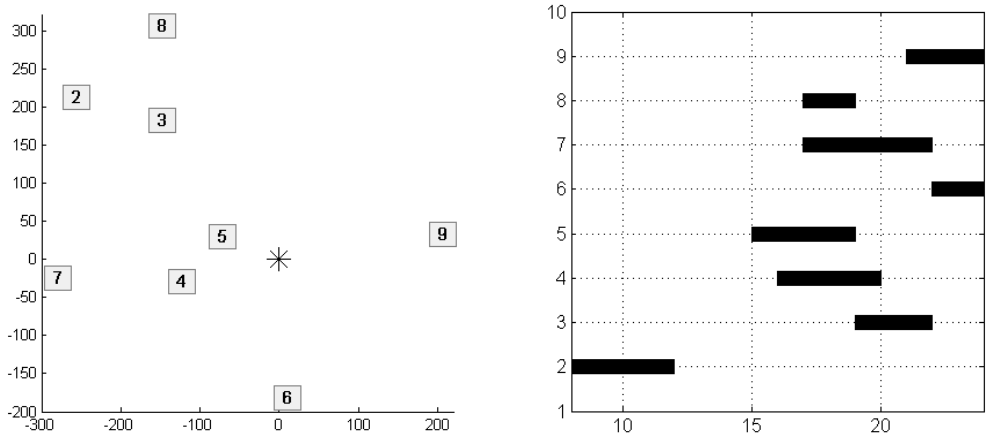

Figure 1: Sample VRPTW with 8 customers

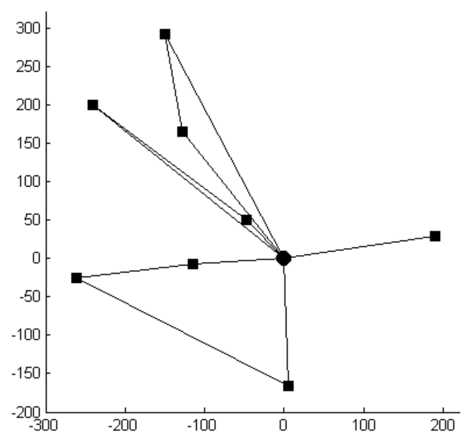

Figure 2: Optimal solution for the sample problem

4. The heuristic

The heuristic is composed of the followings steps:

Step 1: Sort the customers according to the time windows. If two customers have the same starting time, put the one with a later closing time higher in the order. Figure 3 shows the result of applying this step to the sample problem above (sorted and relabeled customers). Note that here customers 5 and 6 have the same starting time but customer 6 has later closing time so it is placed higher in the order.

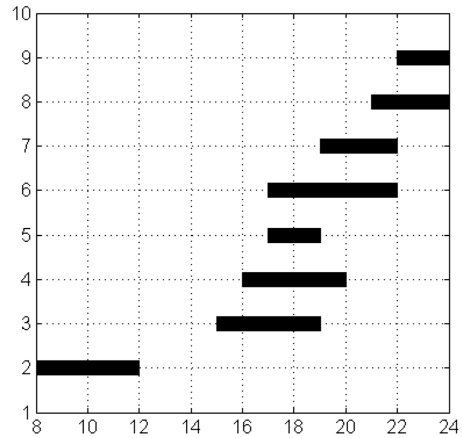

Figure 3: Customers sorted and relabeled according to time windows

Step 2: Calculates the savings according to the savings algorithm (Clarke and Wright, 1964). Savings are calculated for a pair of customers *i* and *j* by the following relation:

$$sv_{ij} = D_{1i} + D_{1j} - D_{ij}$$

Table 1 shows the top 10 pairs in terms of savings. What the data in this table means is that if we join customers 2 and 5, we will have the highest saving possible, and thus joining 2 to 5 is more beneficial to joining 2 to 7 or any other customers.

Saving	**595**	**386**	**372**	**327**	**253**	**239**	**228**	**171**	**155**	**141**
Customer *i*	2	2	5	2	5	6	4	2	4	4
Customer *j*	5	7	7	6	6	7	6	4	7	5

Table 1: Top 10 pairs in terms of savings

Step 3: Starting from the top of the list, we go over each pair and attempt joining them such that they are visited through the same route. The following rules are applied when joining customer pairs:

- If both customers are on non-trivial routes they cannot be joined. A non-trivial route is one that visits more than one customer.
- Joining a pair effectively merges their current routes (either two trivial routes or one trivial with one non-trivial route), so the new merged route must meet problem constrains on maximum driving and working times and also capacity.

- The order by which customers in the new route are visited must be feasible in terms of customer time windows. This requires time window calculations every time a route is being extended.

Exactly how the sequence of visiting the customers should be when a route is being extended by a new customer is the source of variety in our heuristic. The simplest way (which is used here in this example) is to use the time window order obtained from Step 1, so that the sequence of visiting the customers is always matched with the order of customers in Figure 3. If such sequence was not feasible, extension will not happen and the pair will be discarded. This basic version of the heuristic actually could obtain the exact same result as the mathematical model (which is shown in Figure 2) on the sample problem.

A more advanced variation of the heuristic tests different visiting sequences by first forming a local cluster out of all the customers of the new route and then running a quick travelling salesman heuristic algorithm on the cluster to find some candidate optimal routes to test. This variation of the heuristic is even more powerful that the first one, even though it is slightly slower. However, the basic version still performed reasonably well on our test cases as the next section will discuss.

5. Results and discussion

Table 2 shows the results of applying the two variations of our heuristic to 6 problem cases with different size and different properties. Convergence time is also given for comparison (values in parentheses are best-so-far values).

	Problem	**Sample**	**2-1**	**2-2**	**2-3**	**3**	**4**
	Customers	8	20	20	20	30	40
	Std. TW Span (hr)	1.061	1.832	1.832	1.832	1.676	1.227
Performance	OPL/CPLEX	2499	4332	4320	3831	4135	(5221)
	Basic heuristic	2499	4541	4375	4155	4557	6213
	Advanced heuristic	2499	4405	4345	4155	4346	5874
	Best difference (%)	*0*	*1.68*	*0.54*	*8.45*	*5.10*	*12.5*
Time	OPL/CPLEX	1 sec.	52 min.	10 sec.	38 min.	328 min.	(650 min)
	Basic heuristic (s.)	0.048	0.061	0.060	0.050	0.051	0.058
	Adv. heuristic (s.)	0.055	0.115	0.124	0.101	0.128	0.161

Table 2: Results and convergence times of the heuristics vs. the mathematical model

Note that the heuristic results are reasonably close to the optimum in the majority of cases, and at worst they are about 12.5% off. The convergence time is also as expected for a heuristic, for it calculated a solution for each case in less than a millisecond on our machine, while the OPL/CPLEX took as much as 5 hours to converge on the same machine. We were not able to make the model converge for a problem of size 40 or more after more than 10 hours. Note that performance seems to have a decreasing trend as problem size increases, which is likely to be due to the raising complexity of the problem space. Also note that performance is slightly unstable, which is a problem that requires further investigation.

We believe that the main reason behind our heuristic's good performance on these problem cases is that when the time windows are tight, the options present in the problem for choosing the order of customers to visit will be limited. For example in the sample problem of Section 3, if a route is to visit both customers 4 and 6, it must be going to 4 first and then to 6. The other direction is simply impossible. This makes joining according to time windows order a reasonably accurate estimation, which is what out heuristic does. As the time windows expand wider, the problem approaches a VRP without time windows, for which the routes are not directed, and the number of possibilities is high.

6. Conclusions and future work

The heuristic proposed in this paper is reasonably good when time windows are tight (with a standard deviation of less than 2 hours). The algorithm is able to find solutions for problems of size 40 in milliseconds, compared to several hours and no convergence for the mathematical model. Nevertheless, the results do not seem to be very stable and performance seems to be dropping with increasing problem size. Dealing with these issues requires further investigations into the mechanisms of the algorithm and nature of the problem space.

There are some areas of potential improvement as well. For example if customers are pre-clustered based both on their geographical positions (conventional clustering) and time windows, we might be able to save some computation and further increase the possibility of finding the best customers to group into the same route. The computational feasibility of testing all possible permutations of customers in a route, upon extension of the route, can also be investigated. If computation cost is acceptable, performance is likely to improve.

Finally, to make the problem more realistic, vehicle speed can be adjusted according to need, as a real-world driver would. When there is hurry, the driver may speed up, and when there is time to kill, the driver may slow down. The effect of problem parameters, especially demand on heuristic performance should also be investigated. We believe that as customer demand average increases with respect to vehicle capacity, performance will gradually decrease and then increase again.

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